FEEDBACK CONTROL OF PROBABILITY AMPLITUDES FOR TWO-LEVEL ATOM IN OPTICAL FIELD

Saifullah *

* *School of Mathematical Sciences*
Government College University
Lahore – Pakistan
Email: saifullahkhalid75@yahoo.com

Abstract: We demonstrate the possibility to stabilize the probability amplitude of the upper level for a single quantum two-level atom in a classical optical field with feedback control scheme.

Keywords: Two-level atom, optical field, feedback control.
PACS: 42.50.-p, 02.30.Yy

1. INTRODUCTION

The methods of feedback control are widely used in the modern physics, but still they are not very popular in quantum optics. Very often this "cybernetical" approach does not demand involvement in very complicated physical devices and can be arranged in a trivial nonlinear system [1].

We apply this technique to control the energy of a two-level atom in the optical external field $E(t)$ in the frame of the so-called "semi classical model" of the atom–field interaction that describes a single quantum two-level atomic system (all other levels are neglected) with classical electromagnetic field.

Recently other authors studied the control of two-level atoms in the frame of open loop-ideology when the controlling field was known *a priori*. It allowed obtaining the different forms of atomic energy spectra, producing π- and $\frac{\pi}{2}$- pulses [2], taking special non-constant shapes of external field [3] etc.

The main feature of the model proposed in this article is that, it is based on the closed-loop approach. This means that we do not initially define the dependency of the field on time, but restore this function for every moment from the current values of the probability amplitudes of the atomic ground and excited levels.

The closed-loop (feedback) scheme for the interaction of two-level atom with external field can be realized in different models. The most famous and fully developed is the approach based on master equations in its both main variants: the Markovian feedback model [4] and the so called Bayesian feedback [5] (the later model was proposed by Wiseman in his comparative analysis of both these models in [6]). The Bayesian ideology is more closely related to our approach because this closed-loop control is constructed directly on the estimation of the system state.

Another approach is to construct the control scheme for a single atom for the quantum control field [7]. In this paper we discuss the classical control field and we do not apply special restrictions on its shape i.e, the optical field shouldn't be sinusoidal as in [8] or have other special time dependency. Thus, our scheme of classical feedback proposed here is similar to the traditional variant of control theory in the form of speed-gradient (SG) method [9], when input variables change proportionally to the speed-gradient of

appropriate goal function. We use the standard notations following [10], but in our model the optical field plays the role of a control signal $u(t)$ for closed-loop or feedback control scheme.

For this purpose we use the real positive goal function Q, measuring how far at the moment we are from the desired state of the atom. As a result, we calculate the control signal $u(t)$, i.e. we restore the shape of the electromagnetic filed $E(t)$ to keep the atom at the upper level.

In the second section of this work, we construct the feedback control model for the single two-level atom in external controlling optical field. Then, in the third section, we apply feedback speed gradient scheme to the non-decay case.

2. TWO-LEVEL ATOM IN CONTROL OPTICAL FIELD

Let's consider the interaction of an optical field $E(t)$ linearly polarized along the x-axis with a two-level atom.

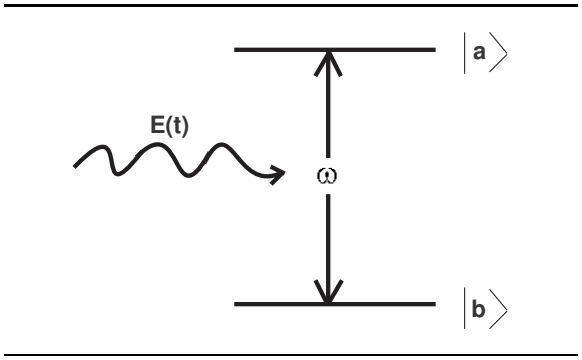

Fig. 1. Interaction of a single two-level atom with an optical field.

Let $|a\rangle$ and $|b\rangle$ represent the upper and lower level states of the atom, i.e. they are eigenstates of the unperturbed part of the Hamiltonian $\hat{H}_0$ with the eigenvalues: $\hat{H}_0|a\rangle = \hbar\omega_a|a\rangle$ and $\hat{H}_0|b\rangle = \hbar\omega_b|b\rangle$. The wave function of a two-level atom can be written in the form

$$|\psi(t)\rangle = C_a(t)|a\rangle + C_b(t)|b\rangle,$$

where C_a and C_b are the probability amplitudes of finding the atom in states $|a\rangle$ and $|b\rangle$, respectively. The corresponding Schrödinger equation is:

$$|\dot{\psi(t)}\rangle = -\frac{\iota}{\hbar}\hat{H}|\psi(t)\rangle,$$

with $\hat{H} = \hat{H}_0 + \hat{H}_1$, where $\hat{H}_0$ and $\hat{H}_1$ represent the unperturbed and interaction parts of the Hamiltonian, respectively [10]:

$$\hat{H}_0 = \hbar\omega_a|a\rangle\langle a| + \hbar\omega_b|b\rangle\langle b| \ ;$$
$$\hat{H}_1 = -\Big(\wp_{ab}|a\rangle\langle b| + \wp_{ba}|b\rangle\langle a|\Big)E(t),$$

where $\wp_{ab} = \wp^*_{ba} = e\langle a|x|b\rangle$ is the matrix element of the electric dipole moment. We neglect the decay of the levels. We express the electric field as

$$E(t) = E_0 u(t),$$

where E_0 is the amplitude and $u(t)$ is the dimensionless control signal. The equations of motion for the amplitudes C_a and C_b may be written as

$$\dot{C_a} = -\iota\omega_a C_a + \iota\Omega_R u(t) e^{-\iota\phi} C_b;$$
$$\dot{C_b} = -\iota\omega_b C_b + \iota\Omega_R u(t) e^{\iota\phi} C_a,$$

where the "Rabi frequency" is defined as $\Omega_R = \frac{|\wp_{ba}|E_0}{\hbar}$, and ϕ is the phase of the dipole matrix element $\wp_{ba} = |\wp_{ba}|e^{\iota\phi}$.

To solve for C_a and C_b, we write the equations of motion for the slowly varying amplitudes as:

$$c_a = C_a e^{\iota\omega_a t} \ \ ; \ \ c_b = C_b e^{\iota\omega_b t},$$

then

$$\dot{c_a} = \iota\Omega_R u(t) e^{-\iota\phi} c_b e^{\iota\omega t} \ ;$$
$$\dot{c_b} = \iota\Omega_R u(t) e^{\iota\phi} c_a e^{-\iota\omega t} \ ,$$

where $\omega = \omega_a - \omega_b$ is the atomic transition frequency. The phase ϕ can be excluded from the system, if we put $\widetilde{c_b} = c_b e^{-\iota\phi}$:

$$\dot{\widetilde{c_b}} = \iota\Omega_R u(t) e^{-\iota w t} c_a$$

Later for simplicity we will denote $\widetilde{c_b}$ with c_b, then finally:

$$\dot{c_a} = \iota\Omega_R u(t) e^{\iota w t} c_b \tag{1}$$

$$\dot{c_b} = \iota\Omega_R u(t) e^{-\iota w t} c_a \tag{2}$$

Now let's suppose that we have the initial conditions:

$$c_a(0) = 0 \ \ ; \ \ c_b(0) = 1 \tag{3}$$

and our goal is to stabilize the atomic system at the upper level: $|c_a|^2 = 1$.

3. SPEED GRADIENT METHOD FOR PROBABILITY AMPLITUDES CONTROL

We have not yet specified the time-dependent function $u(t)$. To find it, we apply the speed gradient (SG) method [9] to control the system behavior.

In this approach, the control action is chosen in the maximum descent direction for a scalar goal function.

The goal in the control process is a smooth scalar function Q with the limit relation

$$\lim_{t\to\infty} Q(x(t), t) \to 0.$$

The purpose of the SG method is to minimize the goal function

$$Q = \frac{1}{2}\Big(|c_a|^2 - 1\Big)^2, \tag{4}$$

where $|c_a|^2 = c_a c_a^*$.

SG represents the control signal u with the time derivative of the goal function Q.

The underlying idea of SG method is that moving along the anti-gradient of the speed $\dot{Q}$ provides decreasing of the goal function. In our case the control signal space is 1-dimensional, thus we reduce our gradient to the partial derivative with respect to u. In the case of proportional feedback with some positive coefficient Γ, it is defined in the form:

$$u = -\Gamma \frac{\partial \dot{Q}}{\partial u} \tag{5}$$

Thus

$$u(t) = \iota \Gamma \Omega_R \Big(|c_a|^2 - 1\Big)\Big(e^{-\iota w t} c_a c_b^* - e^{\iota w t} c_b c_a^*\Big) \tag{6}$$

Putting value of $u(t)$ from Eq.(6) in Eqs.(1) and (2), we have the following system of equations:

$$\dot{c}_a = \Gamma \Omega_R^2 \Big(|c_a|^2 - 1\Big)\Big(e^{2\iota w t} c_a^* c_b^2 - c_a |c_b|^2\Big) \ ;$$
$$\dot{c}_b = \Gamma \Omega_R^2 \Big(|c_a|^2 - 1\Big)\Big(c_b |c_a|^2 - e^{-2\iota w t} c_a^2 c_b^*\Big).$$

Now suppose that

$$\rho_a = c_a c_a^* = |c_a|^2 \ \ ; \ \ \rho_b = c_b c_b^* = |c_b|^2 \ \ ;$$
$$\iota \rho_- = e^{-\iota \omega t} c_a c_b^* - e^{\iota \omega t} c_a^* c_b \ \ ;$$
$$\rho_+ = e^{-\iota \omega t} c_a c_b^* + e^{\iota \omega t} c_a^* c_b.$$

Hence we have the following four equations:

$$\dot{\rho_a} = 2\Gamma \Omega_R^2 \Big(\rho_a - 1\Big)\Big[\Big(\frac{\rho_+^2 - \rho_-^2}{4}\Big) - \rho_a \rho_b\Big] \ ;$$
$$\dot{\rho_b} = 2\Gamma \Omega_R^2 \Big(\rho_a - 1\Big)\Big[\rho_a \rho_b - \Big(\frac{\rho_+^2 - \rho_-^2}{4}\Big)\Big] \ ;$$
$$\dot{\rho_+} = \omega \rho_- \ ; \tag{7}$$
$$\dot{\rho_-} = -2\Gamma \Omega_R^2 \Big(\rho_b - \rho_a\Big)\Big(\rho_a - 1\Big)\rho_- - \omega \rho_+ \ .$$

Also from Eq.(6) the control signal $u(t)$ becomes

$$u(t) = -\Gamma \Omega_R \Big(\rho_a - 1\Big)\rho_- \tag{8}$$

With initial conditions $\rho_a(0) = 0$, $\rho_b(0) = 1$ we have

$$\dot{\rho_a} + \dot{\rho_b} = 0 \ ,$$

that means in fact:

$$|c_a(t)|^2 + |c_b(t)|^2 = 1,$$

which is the simple statement that the probability to find the atom in one of its states $|a\rangle$ or $|b\rangle$ is 1.

Thus, we can simplify the system (7), putting $\rho_b = 1 - \rho_a$.

The system (7) has two equilibrium (fixed) points:

$$(\rho_a, \rho_+, \rho_-) = (0,0,0), (1,0,0)$$

On the Figs. 2,3 we demonstrate the result of our control procedure for: $\Gamma = 0.1\ sec$, $\Omega_R = 10^2\ sec^{-1}$ and $\omega = 10^3\ sec^{-1}$.

On Fig.2 we show the solution of Eq.(7a).

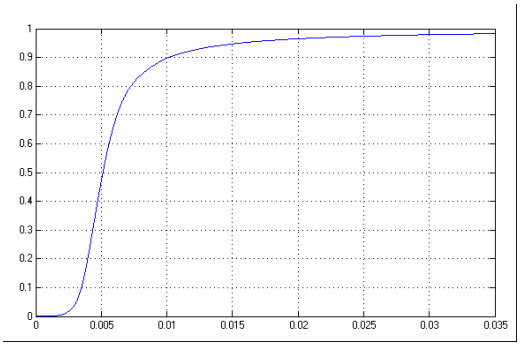

Fig. 2. The density matrix element $\rho_a(t)$ for the control procedure (4)-(5)

.

On Fig.3 we show the control signal(8).

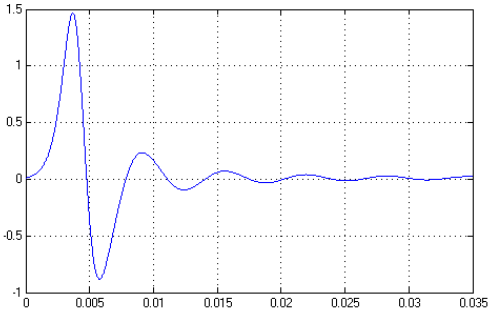

Fig. 3. The control signal $u(t)$ for the system (7).

4. CONCLUSION

The SG algorithm can be easy applied to establish feedback control for the probability amplitudes of two-level atom.

This scheme can be modified if we take into consideration the decay of the atom levels, because in this case the goal $Q = \frac{1}{2}\Big(|c_a|^2 - 1\Big)^2$ is not achievable for SG algorithm in principle. For this purpose we will redefine the goal function Q.

5. ACKNOWLEDGEMENT

I wish to thank the reviewer(s) for their valuable comments and suggestions, which significantly improved the paper.

I am also thankful to Dr. Sergei Borisenok (Department of Physics, Herzen State University, Saint Petersburg, Russia) for productive scientific discussions, valuable suggestions and guidance.

REFERENCES

[1] A. Fradkov. Physica D 128 (1999) 159168.

[2] N. Imoto, Prog. Cryst. Growth Charact. Mater. 33 (1-3) (1996) 295.

[3] A.Di. Piazza, E. Fiordilino, M. H. Mittleman. J. Phys. B: At. Mol. Opt. Phys. 34 (2001) 3655.

[4] H.M. Wiseman, G.J. Milburn. Phys. Rev. Lett. 70 (1993) 548.
H.M. Wiseman, G. Milburn. Phys. Rev. A 49 (1994) 1350.

[5] A.C. Doherty, K. Jacobs. Phys. Rev. A 60 (1999) 2700.
A.C. Doherty, S. Habib, k. Jacobs, H. Mabuchi, S.m. Tan. Phys. Rev. A 62 (2000) 012105.

[6] H.M. Wiseman, Stefano Mancini, Jin Wang. Phys. Rev. A 66 (2002) 013807.

[7] C.K. Law, J.H. Eberly. Phys. Rev. Lett. 76 (1996) 1055.

[8] J.A. Dunningham, H.m. Wiseman, D.F. Walls. Phys. Rev. A 55 (1997) 1398.

[9] A.L. Fradkov, A.Yu. Pogromsky. Introduction to Control of Oscillations and Chaos, Singapore: World Scientific, 1999.

[10] M.O. Scully, M.S. Zubairy. Quantum Optics, Cambridge University Press, 1997.